HEAT AND MASS TRANSPORT IN NONISOTHERMAL TURBULENT FLOW WHEN THERE IS A SCREEN FORMED BY A GAS OF A DIFFERENT NATURE

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We obtain a limiting relative law for heat and mass transport when there is a gas screen in a turbulent boundary layer, which makes it possible to take into account the effect of nonisothermal flow on the turbulent heat and mass transport beyond the region where the foreign gas is injected. The theoretical results are compared with experimental data on the intensity of burn-up of a graphite surface in an air flow when helium is injected through a tangential slit. The experimental data were obtained from the diffusion region of the burn-up.

Most of the investigations on a gas screen were carried out in conditions in which the physical properties of the gas flow were constant. They are basically concerned with determining the effectiveness of the boundary cooling of the adiabatic wall (the determination of the temperature $T_W{}^*$ and the concentration $K_W{}^*$ on the adiabatic part of the wall).

It was assumed that heat and mass transport under nonadiabatic conditions (density of the thermal flux at the wall $q_W \neq 0$) can be calculated by the usual methods if as the typical temperature drop the difference between the wall temperature and the temperature of the adiabatic wall, $T_W - T_W{}^*$ under the given conditions was used.

It was shown in [1-5] that this assumption is valid for a gas flow with constant physical properties. In a nonhomogeneous boundary layer the complete enthalpy of the gas should be used instead of the temperature.

For investigations of a turbulent boundary layer with variable physical properties, similarity conditions for the velocity and temperature (complete enthalpy) profiles are commonly used to determine the density distribution through the thickness. It was shown in [5] that when there is a gas screen the similarity condition holds if the enthalpy (temperature) profiles are constructed with respect to their "equilibrium" values (i.e., with respect to their values at the point of the boundary layer under consideration when there is a screen at the adiabatic wall):

$$\frac{h - h^*}{h_w - h_w^*} = 1 - \omega \qquad (1)$$

Here $\omega = w_x/w_0$, w_x is the velocity in the boundary layer, w_0 is the velocity of the basic flow, h is the gas enthalpy, h_W and $h_W{}^*$ are the gas enthalpy at the wall with ($q_W \neq 0$) and without ($q_W = 0$) heat transfer, h^* is the gas enthalpy at the point of the boundary layer under consideration when there is a gas screen at the adiabatic wall ($q_W = 0$).

It is assumed that the x axis is directed along the wall and that the y axis is perpendicular to it. The subscripts w, s, the asterisk and affix zero denote respectively parameters at the wall, in the slit, adiabatic conditions at the wall and conditions in the basic flow.

When there is boundary cooling of the adiabatic wall $(\partial h/\partial y)_W = 0$, as a result of turbulent mixing the heat content of the gas inside the boundary layer becomes uniform. The greatest intensity of turbulent mix-

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ing occurs directly at the wall in the region of large velocity gradients $\partial w_x/\partial y$. Hence the region with $(\partial h/\partial y) \approx 0$ is strongly washed away from the wall into the depth of the boundary layer and the region in which $h^* \approx h_w^*$ increases. This process of levelling out the heat content of the gas in the neighborhood of the adiabatic surface can be written symbolically as follows:

$$h^* \to h_w^* \quad \text{as} \quad x \to \infty \tag{2}$$

This assumption was made in [1] in the investigation of boundary cooling. It should be noted that the resulting theoretical equations for the effective thermal screens agree with the experimental data of several authors.

If we use Eq. (2), the similarity of the velocity and enthalpy profiles (1), when there is a screen at the nonadiabatic surface, can be written as

$$\vartheta \equiv \frac{h - h_w}{h_w^* - h_w} = \omega \tag{3}$$

When there is similarity of the velocity and temperature profiles of the form (3), the limiting relative law of turbulent heat and mass transport obtained by Kutateladze and Leont'ev [1,6] preserves its usual form and for a boundary layer with a screen we have

$$\Psi_{R^{**}\to\infty} = \left(\int_0^1 \sqrt{\frac{q_0^\circ}{q^\circ}\frac{\rho}{\rho_0}}\, d\vartheta\right)^2 \tag{4}$$

Here $\Psi = (St/St_0)_{R^{**}}$ is the relative heat-transfer coefficient; St and St_0 are the Stanton numbers in the conditions under consideration and in standard conditions at the same Reynolds number R^{**}, constructed for the energy loss through the thickness; $q^\circ = q/q_w$ is the relative density of the thermal flux at the point of the boundary layer under consideration; q_0° is the relative density in standard conditions; ρ/ρ_0 is the relative gas density.

We take the turbulent boundary layer on a smooth impermeable surface in a quasi-isothermal flow of an incompressible gas (with constant physical properties) as the standard conditions.

To determine the relative equations for heat and mass transport from (4) we have to know the thermal flux distribution and the gas density over a cross section of the boundary layer.

The approximate profile of the thermal flux over the cross section of the boundary layer when there is a gas screen has the same form as when there is no screen [1,6]:

$$q^\circ/q_0^\circ = 1 + b_1\vartheta \tag{5}$$

where $b_1 = j_w/\rho_0 w_0 St$ is the permeability parameter of the wall, j_w is the transverse mass flux at the wall.

The density distribution across the cross section of the boundary layer is determined from the equation of state of an ideal gas

$$\frac{\rho}{\rho_0} = \frac{MT_0}{M_0 T} \tag{6}$$

and from the condition of similarity of the profiles of the total enthalpies and concentrations

$$\vartheta = \frac{h - h_w}{h_w^* - h_w} = \frac{K_i - (K_i)_w}{(K_i)_w^* - (K_i)_w} \tag{7}$$

Here M and M_0 are the molecular weight of the gas at the point of the boundary layer under consideration and in the unperturbed flow; K_i is the weighted concentration of the i-th component of the gaseous mixture.

The temperature at the point under consideration in the subsonic gas flow is

$$T = \frac{h}{c_p} = \frac{h_w + (h_w^* - h_w)\vartheta}{c_p} \tag{8}$$

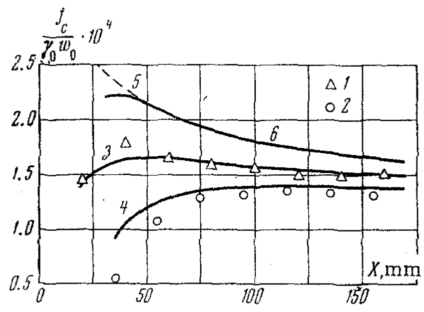

Fig. 1

The specific heat and molecular weight of the gaseous mixture are determined from the equations

$$c_p = \sum_1^n c_{p,i} K_i, \quad \frac{1}{M} = \sum_1^n \frac{K_i}{M_i} \tag{9}$$

Here $c_{p,i}$ is the specific heat of the i-th component.

If we ignore the relation between the specific heat and the temperature, from (7) and (9) we obtain

$$c_p = c_{p_w} + (c^*_{p_w} - c_{p_w})\vartheta, \quad \frac{1}{M} = \frac{1}{M_w} + \left(\frac{1}{M_w{}^*} - \frac{1}{M_w}\right)\vartheta \tag{10}$$

Here c_{p_w} and M_w are the specific heat and molecular weight of the gaseous mixture at the wall when there is heat and mass transport, $c^*_{p_w}$ and $M_w{}^*$ are the specific heat and molecular weight of the gaseous mixtures at the adiabatic wall.

Substituting (8) and (10) in the equation of state (6), we can obtain the density distribution across the subsonic boundary layer in the following general form:

$$\frac{\rho}{\rho_0} = \frac{M_w{}^*}{M_0}\frac{T_0}{T_w{}^*}\frac{[\psi_2 + (1-\psi_2)\vartheta]}{[\psi_1 + (1-\psi_1)\vartheta][\psi_3 + (1-\psi_3)\vartheta]}$$

$$\left(\psi_1 = \frac{h_w}{h_w{}^*}, \quad \psi_2 = \frac{c_{p_w}}{c^*_{p_w}}, \quad \psi_3 = \frac{M_w{}^*}{M_w}\right) \tag{11}$$

As we see from (11), the density distribution depends on the parameters of the screen through the molecular weight and temperature of the gas at the adiabatic wall.

In the case of a quasi-isothermal flow past the surface $T_w \approx T_w{}^* \approx T_0$, the density distribution depends only on the composition of the mixture (it is determined only by the molecular weight)

$$\frac{\rho}{\rho_0} = \frac{M_w{}^*}{M_0}\frac{1}{\psi_3 + (1-\psi_3)\vartheta} \tag{12}$$

For gases of the same atomicity we have $c_p M \approx$ const, or

$$\frac{c_{p_w}}{c^*_{p_w}} = \frac{M_w{}^*}{M_w}, \quad \psi_2 = \psi_3 \tag{13}$$

Then the density distribution for a nonisothermal subsonic boundary layer has the form

$$\frac{\rho}{\rho_0} = \frac{M_w{}^* T_0}{M_0 T_w{}^*}\frac{1}{\psi_1 + (1-\psi_1)\vartheta} \tag{14}$$

The limiting form of (4), after substituting in it the profiles of the thermal fluxes (5) and the density distribution (14), is

$$\Psi = \frac{M_w{}^* T_0}{M_0 T_w{}^*}\left(\int_0^1 \frac{d\vartheta}{\sqrt{[\psi_1 + (1-\psi_1)\vartheta](1+b_1\vartheta)}}\right)^2 \tag{15}$$

Integrating this with $b_1 = 0$ (impermeable surface), we obtain the effect of the absence of isothermal conditions on turbulent heat and mass transport when there is a gas screen

$$\Psi_T = \frac{M_w{}^* T_0}{M_0 T_w{}^*}\left(\frac{2}{\sqrt{\psi_1} + 1}\right)^2 \tag{16}$$

Taking account of (13), we can write (16) as

$$\Psi_T = 4\left(\sqrt{\frac{h_w}{h_0}} + \sqrt{\frac{h_w{}^*}{h_0}}\right)^{-2} \tag{17}$$

As we see (16) and (17), when there is no screen ($h_w{}^* \to h_0$, $M_w{}^* \to M_0$) become the familiar equation due to Kutateladze [7]. The additional factor in front of the parenthesis in (16) and the second term in the denominator of (17) make it possible to take into account the presence of a gas screen in the nonisothermal flow past the surface. The quantities M_W^*, T_W^*, and h_W^* are determined by the effectiveness of the gas screen on the adiabatic surface [1-4,8].

Figure 1 compares the theoretical results with the experimental data obtained by the author in conjunction with E. I. Sinaiko. The data are for experiments on the burn-up of a graphite surface in an air flow with injection of helium through a tangential slit. The experimental apparatus, the experimental method, and the treatment of the data are described in detail in [9]. The density of the graphite under test was 1895 kg/m^3. The temperatures of the injected helium was equal to the air temperature, $T_S = T_0 = 290°K$, the temperature of the graphite wall was $T_W = 1950°K$. The width of the slit was s = 2.08 mm. The mass air flow was $\rho_0 w_0 \approx 150\ kg/m^2 \cdot sec$. The experimental points 1 were obtained with an injection parameter $\rho_S w_S/\rho_0 w_0 = 0.041$, and the points 2 with the parameter $\rho_S w_S/\rho_0 w_0 = 0.073$.

As we see from [9], when there is an inert gas screen the intensity of burn-up of the graphite surface can be determined from the equation

$$\frac{j_c}{\rho_0 w_0} = 21.8 \cdot 10^{-3} (K_0)_w{}^* R_x^{-0.2} S^{-0.6} \left(\frac{\mu_w}{\mu_0}\right)^{0.2} \Psi^{0.8} \tag{18}$$

Here $(K_0)_W{}^*$ is the oxygen concentration at the wall in the absence of chemical reaction, S is the Schmidt number, μ_W and μ_0 are the gas and basic flow viscosities. Figure 1 shows the curves calculated from (18) and (16), 3 and 4, in the conditions of the experiment. The curves 5 and 6 were calculated without taking the factor $M_W{}^* T_0/M_0 T_W{}^*$ in (16) into account. It was assumed in the calculations that the Schmidt number was equal to the Prandtl number.

It follows from the comparison that there is satisfactory agreement between theory and experiment if the factor $M_W{}^* T_0/M_0 T_W{}^*$ in the limiting equation (16) is taken into account.

Thus, we have shown that we have to take the presence of a gas screen into account in determining the relative laws of heat and mass transport.

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